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(Vol. VII.—April, 1878.)

DISCUSSIONS

ON THE THEORETICAL RESISTANCES OF RAILROAD CURVES.*

MR. O. CHANUTE.—It is interesting, but perhaps misleading to note that Mr. Whinery, from purely theoretical considerations, has arrived almost exactly at the amount of curve resistance for cylindrical wheels, which has been ascertained to exist, by experiment, upon ordinary wheels, at low speed.

As the resistance depends upon the number of degrees of curvature traversed, and as all the elements enumerated by the author vary with the amount of sharpness of the curve, with one exception, and the latter is of small relative importance, we may reduce these elements to factors of the degree of curvature, and they will then be as follows, for each degree of angular deflection per hundred feet :

Resistance of curves in pounds per ton per degree, 4 feet 8½ gauge :

Twist of wheel, or rotation on itself.....	0.0010 lbs.
Slip of wheel on shorter inner rail	0.1713 “
Flange friction to change direction.....	0.2450 “
Loss of power at couplings, from.....	0.0020 to 0.0213 “

Total resistance per degree..... 0.4386 lbs.

The last element alone not varying directly with the degree of curvature.

We may assume, therefore, that the result of the investigation is, that curves add about one-half pound per degree to the resistance on straight lines. Some additional elements being stated to have been left out.

* Paper CLIX, Vol. VII.

In 1854, Zerah Colburn satisfied himself in a series of experiments on the 6 feet gauge of the Erie Railway, that the curve resistance was about one-half pound per ton per degree, and this factor has been used upon this line ever since, in calculating curve resistances.

In 1844, Mr. B. H. Latrobe made a series of 36 experiments on curve resistance, on the 4 feet 8½ inch gauge of the Baltimore and Ohio Railroad. The average result of these experiments, as stated in Vose's "Manual for Railroad Engineers," was, that the resistance was 6.80 pounds per gross ton of 2 240 pounds, on a straight, level line, and 13.22 pounds per gross ton on a curve of 400 feet radius, or an angular deflection of 14° 19½', at speeds varying from 1½ to 3½ miles per hour.

The resistance, therefore, was as follows :

	Per Gross Ton.	Per Net Ton.
On curve 14° 19½'	13.22 pounds	11.81 pounds.
On straight line	6.80 "	6.07 "
Increased resistance	6.42 pounds,	5.74 pounds.
Increased resistance per degree	0.4488 "	0.4008 "

Or almost exactly the same as calculated in the paper of Mr. Whitney.

Very recent experiments (May, 1878), upon the Metropolitan Elevated Railroad in New York, have shown an increased resistance produced by a curve of 90 feet radius (63° 40') of 350 pounds for a single car of 10 tons, and of 1 100 pounds for a train of four cars, weighing 40 tons, both with fixed wheels, and on a gauge of 4 feet 8½ inches. Thus, in the first case the curve resistance was 0.5497 pounds per ton per degree, and 0.4319 pounds per ton per degree in the second.

The latter experiment having also included a train with loose wheels, it may be interesting to give the details. Each train consisted of four cars, weighing 40 tons, one with ordinary wheels fixed on the axle, and the other with loose wheels. The curve was of 90 feet radius.

	Rigid Wheels, 40 Tons.	Loose Wheels, 40 Tons.
Traction on curve (63° 40')	1 700 pounds	1 300 pounds.
" " straight line	600 "	450 "
Increased resistance	1 100 pounds	850 pounds.
Increased resistance per ton	27.50 "	21.25 "
Curve resistance per ton per degree	0.4319 "	0.3337 "

It will be noted that the loose wheels show a less resistance than the fixed wheels, by 22 or 23 per cent., both on curves and on straight lines. The difference on straight lines proves little, the trains being different ;

whether the difference of 150 pounds upon the curve will compensate for the increased complication, future experience must decide. The resistance on straight lines is in both cases too great, being 15 pounds per ton for the fixed wheels, and 11½ pounds per ton for the loose wheels, while at the low speed at which these experiments were tried, it is generally from 4 to 10 pounds per ton on our American railroads.

I regret not to have at hand the record of some English experiments, but I believe that the British practice is to allow about one pound per ton per degree, for the increased resistance on curves.

There is in Spon's "Dictionary of Engineering" a very full account of the numerous experiments made in France in 1862-3 and 4, by three French engineers, Messrs. Vuillemin, Güebhard and Dieudonné, upon the resistance of trains under all conditions of working.

Among other things, they ascertained that at a speed of 15 or 16 miles per hour, a curve of 1 000 metres, or 3 281 feet radius (say 1½ degree curve), added 1 kilogramme per French ton, or 2 pounds per net (American) ton, to the resistance on straight lines. On a curve of 800 metres, radius, or 2 624 feet (say 2° 10') the increased resistance was 3 pounds per net ton. The increased curve resistance, therefore, was 1.1428 pounds per ton per degree in the first case, and 1.3820 pounds per ton per degree in the second.

It must be remembered, however, that these English and French factors were obtained with European four-wheeled cars, whose long, rigid wheel base must develop greater curve resistance than our short and pivoting American truck.

The agreement between the results of experiments, and of the theoretical calculations in the paper under discussion, would be more satisfactory if the latter had been made for conical, instead of cylindrical wheels. The author admits that on a curve of 1 887 feet radius, with coned wheels of the Pennsylvania R. R. standard, the two principal elements of resistance, that from the slip of the inner wheel, and the flange friction in changing the direction (amounting together to 0.4163 pounds per ton per degree), would disappear if the wheels assumed their proper position on the track.

He points out, however, that in order that this shall occur, two conditions must obtain simultaneously: 1st, that the curve shall be of the radius corresponding to the coning of the wheels, and 2d, that the speed shall correspond to that for which the outer rail is elevated.

Not only do these two conditions but seldom occur exactly together in practice, but there is an additional element of resistance, incapable of exact computation, which not only adds considerably to the resistance, but prevents the wheels from adjusting themselves to their proper position on the track, and remaining so around the curve.

This consists of the additional flange friction which results from the oscillations of the truck upon its pivot. These occur upon straight lines, but more especially on curves where the impulse given to the wheel in changing its direction, drives the truck diagonally towards the opposite rail. These oscillations cause the wheel flanges to impinge first on one rail, and then on the other, and thus both add to the resistance, and prevent the coning of the wheels from being continuously effective.

It is not improbable that the constant variations in the resistance of trains, indicated by the dynamometer, are largely caused by this varying flange friction, due to the swaying of the truck from side to side.

An additional element of resistance is due to the fact, that the axles being held rigidly parallel by the framing of the truck, cannot assume a truly radial position to the curve, and hence the tendency of the whole is to roll in a straight line, even when the coning of the wheels brings unequal diameters to bear upon the two rails. It may be doubted, therefore, whether the fourth element of resistance, enumerated by the author, that due to the flange friction requisite to change the direction, is ever eliminated in any position of the wheels on the curve.

It may, however, be noted that the resistance on straight lines due to the coning of the wheels, has been considerably over estimated in the paper under discussion.

The author is quite correct in stating that the part in contact between the wheel and rail, is a surface. The metal, being elastic, yields in both wheel and rail, and the portion in contact, instead of being a line, is a parallelogram more or less irregular in outline. I measured these surfaces some years ago, under the driving wheels of locomotives, and found that they varied from 0.13 to 0.42 of a square inch in area, corresponding to pressures from 85.961 to 26.607 pounds per square inch, the diagrams having been published in the "Railroad Gazette" for April 21st, 1876.

It is an error, however, to assume that for car wheels this parallelogram is anything like one inch in length, transversely of the rail. This was probably arrived at by assuming that the portion of the rail kept bright by the trains, truly represents the length of contact. This bright portion, however, is not only kept so by the action of successive wheels,

many of them much worn, which strike various points on the rail, but is also largely the result of the attrition of the driving wheels of locomotives, which are but slightly coned in the flanged tires, and not all in the blank tires. It does not, therefore, give a true measure of the length of the parallelogram of contact. It is probable that for a new car wheel upon a new rail, it does not exceed $\frac{3}{4}$ of an inch in length, and in fact the bright streak left by the first few trains upon the Elevated Railroad in New York, now about ready for operation, does not exceed $\frac{1}{4}$ to $\frac{3}{8}$ of an inch across the face of the rail.

The error in the assumption of the length of this surface of contact, causes a corresponding error in the calculations of the resistance due to the coning of the wheels, which, instead of being about half a pound per ton on a straight line, is probably not more than $\frac{1}{4}$ to $\frac{1}{2}$ of a pound per ton, or materially less than the curve resistance for a single degree, while the curve resistance, say on a ten degree curve, would be about five pounds per ton.

The paper is so interesting and valuable that it is to be hoped that the author will pursue further his investigations, and give to the society a supplemental paper on the same subject.

MR. CHARLES E. EMERY.—Mathematical discussions are of great value in enabling the well-understood relations of simple physical phenomena to be grouped together to determine approximately questions of a higher order, and the resulting expressions in turn suggest the direction of investigations whereby the constants in the equations may be corrected, and the experiments thereby be comprehensively utilized.

The paper of Mr. Whinery under discussion is of great interest, and shows evidences of thought and careful preparation. In my opinion, however, he has omitted two elements of curve resistance and over-estimated a third, while two other elements, separately treated in his paper, so modify each other that they should be jointly considered. The various modifications to some extent balance each other, but all are essential to a complete examination of the subject.

Both elements believed to be omitted affect principally the flange resistance. The paper discussed only the resistances of this kind arising from the fact that parallel axles in a truck cannot take radial positions on a curve. The wheels tend to roll in a tangent but by flange resistance are continually slipped transversely on the rails. (See § 6.) The paper separately estimates, § 5, the resistance due to slipping the wheels the distance due to the difference of length of the inner and outer rails, but

neglects to include the increased flange resistance caused by the longitudinal slip. The inward slipping due to the parallelism of the axles would occur if the wheels on the two ends of the axle could turn independently one of the other, but the flange resistance necessary to slip the wheels in the direction of the rail would be saved by the use of independent wheels, and should, therefore, be included in ordinary cases where the wheels are rigidly connected to the axle.

These considerations, however, lead us to the conclusion that it is erroneous to consider the longitudinal and transverse slipping separately.

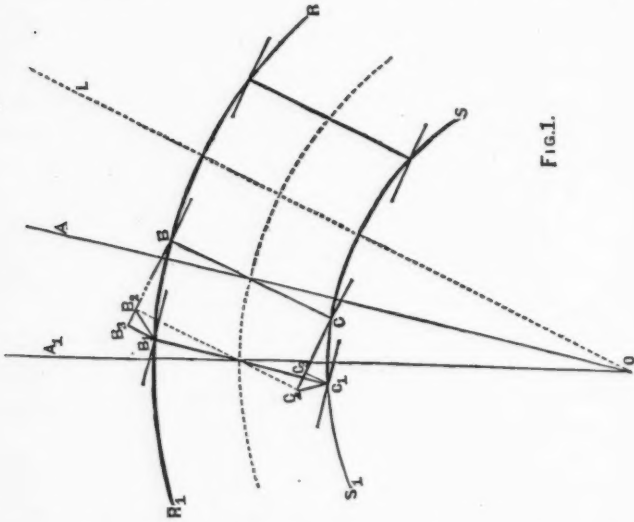


FIG. 1.

Referring to Fig. 1, let RR_1 and SS_1 represent curved rails; OA and OA_1 two radii of the curve, and BC and B_1C_1 the positions of the leading axle of a truck corresponding to such radii, then it is evident that as the system moved through the angle at O , the wheels at B and C would tend to roll in parallel lines to the points B_2 and C_2 , but would be forced to move to the positions B_1 and C_1 . That is, B would be forced *inward* a distance B_2B_1 and *forward* a distance B_2B_2 , so the actual slip would equal the diagonal B_2B_1 . At the same time C would be forced *inward* a distance C_2C_1 , and *backward* a distance C_2C_2 , so the actual slip of the inside wheel would equal the diagonal C_2C_1 . Differences in the distribution of the weight will affect the relative amount of slip on the two rails,

but when the car is uniformly loaded on the two sides and the super-elevation of the outer rail, that due to the speed, the slip should be as per diagram. Corresponding modifications of Mr. Whinery's formula will be submitted when his paper is accessible in print.

In this connection it is suggested also, that probably formula (4), § 4, could also be included in the same expression, as the problem of twisting each wheel, or practically a short cylinder on the rail through a given angle, is much the same as slipping two connected wheels, or practically a long cylinder through the same angle.

The second element of resistance neglected is that required to turn the car through the angle from one tangent to the other. If we suppose a car body supported on a single truck, and propelled on a curve by force applied to the truck itself, the whole mass will develop the centrifugal force discussed in § 1 of the paper. The car body would not, however, change its angle in space, but remain in all positions on the curve parallel with its original position. (See Figure 2.) Under the conditions

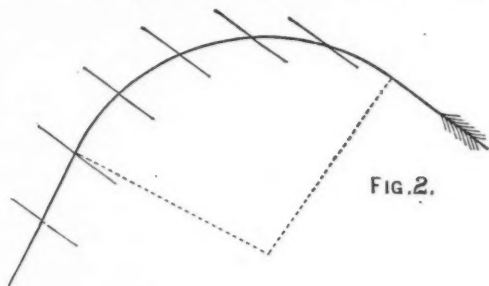


FIG. 2.

of practice, then, force must be applied to revolve the car body through the angle named. If the body be fast to the frame carrying the wheels, as in short cars, the angle of the whole mass must be changed on a centre passing through the rear axle, and the force required to make the change causes extra resistance on the flange of the outer leading wheel. In a car with two trucks each must be revolved by the leading wheels as before, but the car body will be turned by the leading truck, on the after truck as a centre. The leading truck of a locomotive makes high speed possible with increased safety and a reduction of resistance, as the mass is piloted around the curve and its angle changed by force applied well forward of the centre of gyration, in relation to the after rigid wheel base.

Similar advantages are obtained with our American double bogie cars. In cars with the body fast to the frame carrying the wheels, as in street cars, the front wheels generally fall inside the centre of gyration of the mass to be revolved, and with the corresponding loss of leverage the flange resistance is materially increased. The force required to turn the car is in any case evidently too important to be neglected.

I must disagree with the author that the tractive efforts transmitted from one car to another on a curve, are reduced at each car in the proportion of the cosine of the exterior angle of the funicular polygon, to radius. (See § 8.) It is simply the problem of deflecting a force as is done when a rope under strain is wound round a pulley; the train being practically a chain for transmitting power. A lateral force is developed in changing the direction represented approximately by the sine of the angle referred to (or) a fraction of the angle. (See equation 10.) But this force is not expended, except so far as it produces friction on the flanges and the shoulders of the axles.

In respect to coning the wheels, I would state that some years since, when riding at a high rate of speed on the Erie Railroad, I observed that the car ran from side to side of the track, bringing up with a jolt against the flanges. This was attributed at the time to coned wheels, and it is probable that slight curves and other causes were sufficient to start the car in a sidling movement, which the coning of the wheels maintained in accordance with the ordinary principles of vibration. The effect of insufficient elevation of the outer rail, is plainly observable on the Baltimore & Ohio Railroad. The track being regulated for coal traffic, in many places the cars of express trains are thrown violently against the outer rail, and rebounding, are thrown out again and again, producing very unpleasant sensations.

The contradictory results of experiments on curve resistance are probably due, in most cases, to changes of speed on the curve. While the speed is slackening, the dynamometer will show less strain than is due to the actual resistance; the reduction of the *vis viva* of the mass developing a propelling force; while, if the engine is worked harder, for the moment, the indications of the dynamometer will be in excess of the true curve resistance by an amount corresponding to the increase of speed. Some want of correspondence among results with the dynograph I could only explain in this way. No dynamometer experiments can be entirely reliable unless conducted in connection with a velocimeter, by means of

which either the speed may be regulated or proper corrections made afterward in the results. Marking the time at intervals on the dynamometer diagram or a sheet of paper, traversed by the train, is not sufficient. A continuous register of the velocity is necessary.*

MR. EDMUND YARDLEY.—Refinements in calculation to determine the elevation of the outer rail on curves seem to me rather unnecessary, since the element of speed in miles per hour which enters the formula cannot be assumed, but the proper elevation due to the average speed, all things considered, must be determined experimentally.

The following example, covering experience for a number of years, may prove interesting. When an assistant-engineer on the Pennsylvania Railroad at Pittsburgh, in 1859, I gave one of the sub-division foremen, at his request, the figures for the elevation of his curves, merely copying from Henck's table the figures for 30 and 40 miles per hour. On my returning to the Pittsburgh Division, nine years afterwards, he told me that when he put up his curves to the figures given in the table for 30 miles, the trains ran around them to his perfect satisfaction. The figures were as follows: $1^{\circ} 4''$; $2^{\circ} 11''$; $3^{\circ} 14''$; $4^{\circ} 23''$; $5^{\circ} 3''$, &c. His division was the first four miles east of Pittsburgh, the track there has a grade of about 52 feet per mile, and the curves, as I remember, were from 2° to 4° . As it was double track, and I understood him to say he used the same figures for both tracks, we must have had speeds here varying from 10 to 40 miles per hour.

While on this subject, let me suggest that answers to the following questions would be interesting:

1st. What is the true criterion for the proper elevation of a curve? Is it: 1st, that the train shall pass round with the greatest smoothness, or 2d, that both rails should wear equally? My opinion is, that the second is the correct one, the passage of the train being due to correct align-

* In the above connection, I will state that in testing the resistance of street cars recently, I found that the customary springing forward of the horses on approaching a curve entirely vitiated the results, and no satisfactory records could be obtained until the cars were tried on a sharp curve and steep grade, where a hard, steady pull was necessary. A practically uniform velocity of approach was obtained by stopping each car six feet before the beginning of the curve. The experiments appeared to show that the mean tractive effort required to propel a street car at a slow speed on the level, is 11.8 pounds per ton. Wheels 30 inches in diameter; journals $2\frac{1}{2} \times 6$ inches, weight of car, empty, 4 500 pounds. The resistance on a level curve of 40 feet radius was for rigid wheels 53.57 pounds per ton of 2 000 pounds, and for independent wheels 34.03 pounds per ton, making the saving in total resistance, due to the use of the latter, 36.48 per cent. Deducting from each 11.8 pounds the resistance on a straight line, the saving in curve resistance proper was found to be 46.78 per cent.

ment. The foreman above spoken of adopted the first, so that I am in doubt whether to be satisfied with his conclusion or not.

2d. Can you elevate the outer rail sufficiently to keep the flanges of the wheels away from it, so that they will not cut (I* never had it occur, but I am informed that it has happened). If so, what were the elevation and other conditions of the case; were the wheels excessively coned?

MR. EDWARD P. NORTH.—I wish to call attention to the fact that though a single pair of wheels would go around a 3° curve without flange friction, in effect two pairs of wheels are placed in a truck with their axles rigidly parallel, and in that case the system will, on a level surface, advance in a right line, even if both the wheels on one side of the truck are of greater diameter than those on the other side. From which it would seem that aside from the necessities of manufacture and affording a margin for wear, there are no advantages to be derived from coning the wheels,

MR. CHARLES L. McALPINE.—Petersburg and Richmond, Virginia, fell into the hands of the Federal troops near the close of the war.

The base of the latter for many months had been at City Point on the James River.

Early one morning imperative orders were received to run the trains of the United States Military Railways into Petersburg with the least possible delay.

This was done by noon of the same day, under circumstances that would be most interesting, but foreign to the subject under discussion.

This order was followed up by another; that railroad communication with Richmond, should be effected at once.

The railroad bridge at Petersburg, over the Appomattox, had been burned. No connection at that place had ever been made in peace times, and the first examinations showed that heavy excavations, &c., requiring time (for which the department never made allowance), *must* be made, before any reasonable connection with the Richmond line could be effected.

This drove the engineer in charge into the apparently inadmissible curve adopted.

* My experience led me to think it never would occur, since the slow freight trains up the mountain cut away the outer rail on the curves at Kittaning Point.

Contrary to the advice of trackman and bridge builders, a sharp curve was laid out, of more than one hundred degrees, with a radius of fifty feet, on what was to become the main line.

The curve was on trestle work, and the outside posts were framed eight inches longer than the inner ones. The ties were of sound white pine, three inches in thickness, and the rails were double spiked.

Two guard rails were used, also double spiked.

The locomotive engineers generally condemned the bridge and curve at first sight after completion, and a strong prejudice was created against it. But the writer selected the worst curve following engine in the service, the "Government," and ordered her to make the first trial.

Walking backwards, and in front, as this engine slowly made its way, it was easy to perceive her action on this sharp curve.

A little pressure on the outer rail, seemed to drive the wheels (both of the trucks and drivers) down on to the inner rail, and demonstrated practically what had been intended, that the trains must be passed through the curve at greater speed.

Thereafter, when the locomotive men became accustomed to the curve the speed through it was usually from eight to ten miles an hour.

A very large traffic passed over this curve for months afterwards, supplying the armies of occupation in Richmond, and at other points to the southward, and no accident or trouble whatever was experienced at the place in question.

MR. F. COLLINGWOOD.—The question of friction on curves naturally leads to the discussion of the various methods proposed for obviating the difficulty, and in this connection I wish to speak of some recent experience at the East River Bridge. The tramway used for transporting material from the pier on the East River to the anchorage and approach has curves of twenty-nine feet, thirty-four feet, forty-six feet and sixty feet radius, and a gauge of four feet eight and a half inches.

In designing the cars, the axles were made rigid, and wheels loose; and as they are for carrying heavy loads, there are no springs.

While the cars are new, they work very well by giving a little end play to the wheels; but after the holes in the hubs become worn, it is only by giving great freedom in the gauge, and at "points," that we can keep them on the track.

I am satisfied that a much better arrangement would be to have the wheels fixed, and divide the axles in the middle, using either long

sheaves to connect them, or central boxes. This would no doubt lessen the friction materially and be free from the great trouble caused by wear in the hubs. .

Another difficulty experienced is the passage of reverse curves, where there is no tangent between them. With such sharp curves, it is absolutely essential to elevate the outer rail, and the result is that some one of the wheels is lifted free from the track as the car passes from one curve to the other. In the management of the cars, a great deal depends upon the skill of the driver in the proper use of the brake, and keeping the proper line of draft, to overcome the tendency to derailment.

MR. WILLIAM H. PAINE.—In an examination of certain mechanical appliances, intended to obviate the rigid axle, I have observed that the amount of lost motion was so great that a really contrary motion or resistance was the result.